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Millisecond model updating for structures experiencing unmodeled high-rate dynamic events

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ABSTRACT

Real-time control of next-generation active structures that experience unmodeled high-rate dynamic events require an up-to-date numerical model of the structural system. Examples of active structures that experience unmodeled high-rate dynamic events include hypersonic vehicles, active blast mitigation, and ballistic packages. Due to the dynamic environments that these structures operate in it follows that a numerical model of the system be updated on the timescale of less than 10 ms. Furthermore, the requirement for the monitoring of unmodeled high-rate dynamic events means that the proposed model updating technique cannot rely on precalculated data sets or offline training, therefore, the real-time structural updating technique must be capable of learning the state of the structure on-the-fly. This work proposes and validates an algorithmic framework for a millisecond error minimization model updating technique that updates a finite element analysis model of the structural system by minimizing the error between the structure's measured state and a series of parallelized models that are calculated in real-time with the structure as it moves through the high-rate dynamic event. The proposed algorithm is numerically and experimentally validated using an experimental testbed designed to simulate the dynamic events of projectiles in ballistic environments that consists of a cantilever beam and a movable roller support that introduces a continuously changing boundary condition to simulate a change in the structural system (i.e. damage). Experimental results demonstrate that the location of the roller on the testbed could be accurately tracked and updated every 4.04 ms with an accuracy of 2.9% (10.05 mm over a beam of 350 mm) for a standard test profile and that the algorithm could track roller movement through an impact loading. Furthermore, the proposed algorithm demonstrated it was capable of tracking stochastic roller movement with an accuracy of 3.73%. The delay in the estimated roller position caused by the time required to collect a sufficient quantity of vibration data, the need for constant excitation of the structure, and the robustness of the proposed algorithm are discussed.

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1. Introduction

Active structures that are expected to respond to unmodeled high-rate dynamic events in real-time require feedback mechanisms to stabilize and operate these systems. It follows that these real-time feedback mechanisms must learn and adapt very fast to the measured signals, on a timescale of under 10 ms. To enable the realization of these feedback mechanisms, the development of structural model updating techniques capable of providing an updated representation of the system state on a timescale of less than 10 ms are a necessary first step. Examples of active structures that experience unmodeled high-rate dynamics include hypersonic vehicles, active blast mitigation, and ballistic packages [1–4]. Unmodeled high-rate dynamic events cannot rely on precalculated data sets or offline training, therefore, the real-time structural updating technique must be capable of learning the state of the structure on-the-fly. High-rate dynamic events are uniquely characterized by 1) large uncertainties in external loads; 2) high levels of nonstationarities and heavy disturbances; and 3) generation of unmodeled dynamics from changes in system configuration [5]. An effective millisecond model updating technique designed for this specific class of active structures must be capable of converging to the current state of the structure following an unmodeled high-rate dynamic event that may be excited through a variety of unknown input forces applied at unknown locations.

The proposed millisecond model updating algorithmic framework presented in this paper seeks to track the system state of a structure that experiences unmodeled high-rate dynamic events. To achieve this, a set of constraints are adopted for this work: 1) There exist too many system states to rely on precalculated data sets or offline training; 2) the magnitude and location of the input force on the structure is unknown; and 3) changes to the structural system are continuous in nature. A well-designed algorithm capable of tracking a structural system must converge very quickly following an unmodeled dynamic event while operating within the constraints described above and providing an updated system state within the 10 ms constraint.

A number of model updating techniques have been developed for the structural health monitoring of civil and aerospace structures. Finite Element Analysis (FEA) model updating can generally be classified into two groups, frequency-based [6–8] and strain-based [9–11] model updating techniques. However, these model updating techniques are usually executed in an offline manner or on a time-scale of hours to months [12–15] as real-time updating of FEA models is challenging due to the associated computational cost. Compounding the challenge of real-time model updating is the development of observers that track the state of a structure experiencing unmodeled high-rate dynamic events [16,17]. In this context, an observer is a mathematical system that models a real system in order to provide an estimate of the real system's state given measurements of the real system's inputs and outputs. However, sophisticated mathematical systems are often required to formulate an estimator that is capable of operating through noise and uncertainty while tracking time-varying parameters/states and as such are difficult to execute within the requisite timescale of less than 10 ms [18].

Online model updating has been successfully implemented for the updating of non-FEA models used in the real-time hybrid testing and simulation of civil structures. Song et al., [19] demonstrated real-time model updating for a nonlinear one-degree-of-freedom structure modeled with a nine parameter Modified Bouc-Wen Model and demonstrated successful model updating capabilities with an average iteration time of 0.28 ms. Hashemi et al. [20] implemented an online model updating technique for a nine parameter Modified Bouc-Wen Model that utilized an inspector module to determine if the model parameters needed updating based on the error monitored between the model and experimental measurement. Experimental validation was carried out on a two-degree-of-freedom steel frame. Beyond real-time hybrid testing, Downey et al. [21] demonstrated that a simplified artificial neural network could be trained online in real-time to reproduce the dynamics of a cantilever beam. Thereafter, the trained network could be used to estimate acceleration in the beam at unobserved locations. Experimental results demonstrated an average cycle time of 7.1 ms with an error of 1.5% of the system state. These methods, while useful in developing representations of the structural systems, do not generate FEA models of the structures of interests. The development of methodologies for creating up-to-date FEA models will enable further advancements in the field of real-time control of next-generation active structures that experience unmodeled high-rate dynamic events.

This work presents and experimentally validates an algorithmic framework for a millisecond error minimization vibration-based model updating technique that seeks to provide an up-to-date FEA model of the structural system by minimizing the error between the system's measured state (i.e. its resonant frequency) and a series of parallelized models that are calculated in real-time with the structure as it moves through the high-rate dynamic event. To reduce the computational costs related to the solving of FEA models an adaptive model parameter search space is defined that considers the current position of the model in the parameter search space when selecting additional parameter configurations to develop FEA models. This continuous formulation and solving of newly calculated FEA models results in an algorithm capable of tracking a structural system through a dynamic event without having to pre-calculate every possible model configuration. This aspect is important when trying to detect, quantify, and localize the damage in a structural system where the failure modes are not well understood (e.g. a system resisting a blast impact). To enable the selection of appropriate algorithm parameters, this work leverages a formulation for a multi-objective optimization problem [22,23] that is configured to consider both the maximum desired iteration time and the maximum desired error. The contributions of this work are threefold, 1) this work formulates and validates (both numerically and experimentally) an algorithm for real-time structural model updating that does not rely on precalculated data sets or offline training; 2) the proposed algorithm is capable of tracking the state of a structure through a dynamic event while updating the model at the timescale of less than 10 ms; and 3) the proposed algorithm is capable of tracking the system state of a structure experiencing unknown input forces at unknown locations.

2. Background

2.1. Specific challenges associated with high-rate dynamic events

As discussed before, high-rate dynamics are uniquely characterized by 1) large uncertainties in external loads; 2) high levels of nonstationarities and heavy disturbances; and 3) generation of unmodeled dynamics from changes in system configuration [5]. A realistic experimental data set that demonstrates the three key challenges associated with tracking a system through a high-rate dynamic event was generated by Hong et al. [18]. The experiment consisted of an electronics unit housing printed circuit boards and accelerometers (Model 72 manufactured by Meggitt) ruggedized for shock survivability and tested in an accelerated drop tower to create high impact conditions as diagrammed in Fig. 1(a). Data was acquired using a Precision Filters signal conditioner coupled with a 14-bit ADC acquisition card sampling at 1 MS/s (PXI-6133 manufactured by National Instruments). The electronic unit was subjected to four consecutive accelerated drops to simulate four consecutive ballistic penetrations. Fig. 1(b) plots a portion of the results obtained from the top accelerometer (more data is available in reference [18]). This experimental investigation demonstrates all three specific challenges associated with high-rate dynamics: 1) the input forces generated by the drop tower are unknown; 2) the acceleration data reports that high levels of nonstationaries are present in the system; and 3) the unaccounted for changes in the back-to-back tests. These results evidence the specific challenges associated with high-rate dynamic events, in particular, a change in system dynamics occurs very quickly and are altered following each successive impact because of internal damage to the components. It follows that the resulting nonlinear and nonstationary dynamic is very complex to model and that any pre-developed physical representation of the system will not accurately track any unmodeled dynamics generated by a change in system configuration, making the state estimation and prediction task difficult. For this reason, real-time inference-based techniques are incapable of predicting the responses of structures that experience high-rate dynamics.

2.2. DROPBEAR experimental testbed

The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was initially introduced and modeled by Joyce et al. [24] and is presented in Fig. 2(a). The DROPBEAR is a cantilever beam featuring two time-varying, user-controlled parameters: a detachable mass and a continuously movable roller constraint. The mass drop simulates sudden damage to the system whereas the movable roller introduces a nonstationary boundary condition into the system. Both parameters were designed to produce a repeatable and controllable change in the system dynamics that is intended to simulate a change in the structural system (i.e. damage). The nature of these changes provides a level of repeatability that would be unobtainable if system changes were introduced into the beam as cracks or other permanent defects.

For the experimental investigation carried out in this work, a $51 \times 6 \times 350$ mm beam with a single accelerometer (model 393B04 manufactured by PCB Piezotronics) mounted at the free edge of the beam was used. The roller followed a predefined profile that ranged from 48 mm (closest to the fixity) to 175 mm as presented in the inset of Fig. 2. The beam is self-excited by the roller's movements and therefore no extraneous inputs are required, however, this does require an initial input to the beam as annotated in the inset of Fig. 2 to initiate vibrations in the beam. In addition to this initial input, the test profile consists of six square wave inputs of increasing amplitude in addition to six sinusoidal inputs and six impulse inputs. For

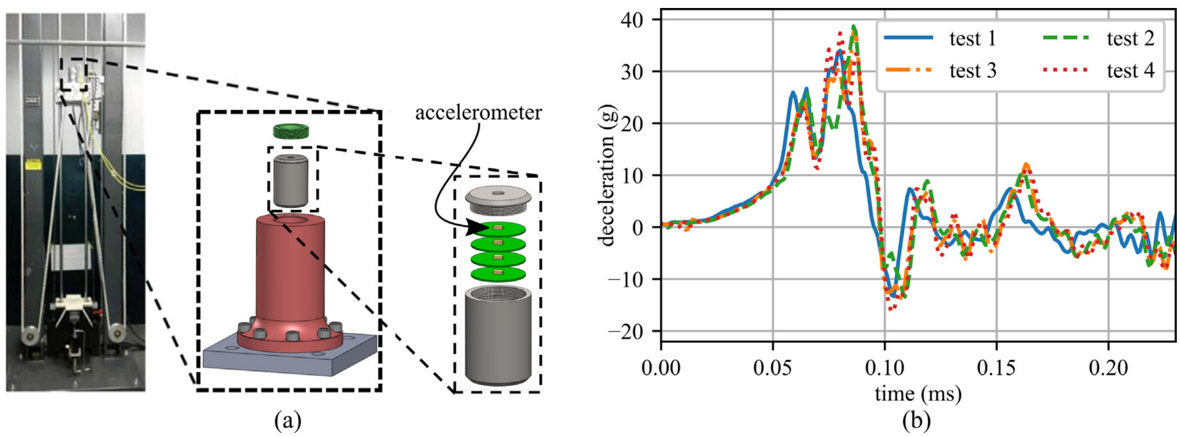

Fig. 1. Variations in responses for a repeated test of a controlled system showing: (a) the structural package with accelerometers under test; and (b) deceleration response of the top accelerometer in four back-to-back shock tests.

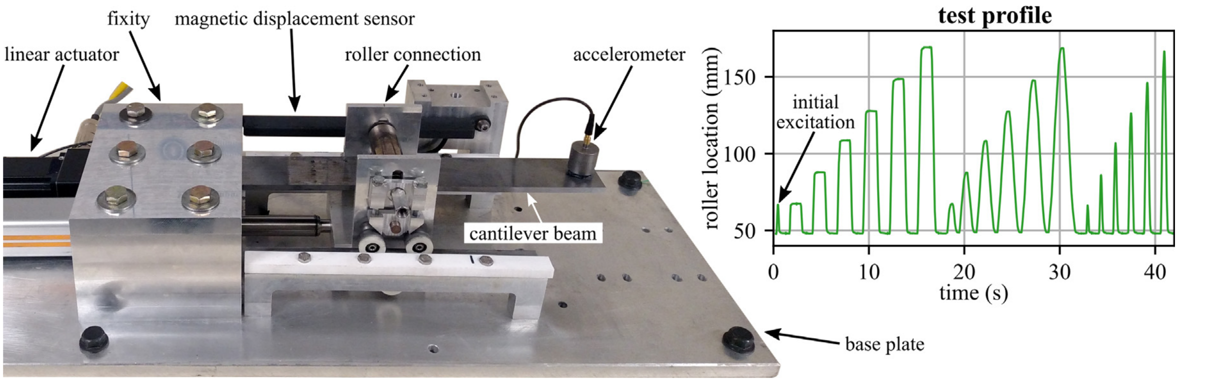

Fig. 2. The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) experimental testbed with key components annotated and the test profile of the roller movement shown in the inset.

the cases of the square and impulse inputs, the actuator velocity was maximized to the extent allowed by the actuator and associated controller (250 mm/s). The detachable mass was not utilized in this work as the focus of the algorithm is the capability to model the nonstationary boundary conditions of the system.

Real-time computations and data acquisition were carried out using a 2.3 GHz eight-core controller (PXIe-8880 manufactured by National Instruments) mounted in a PXI chassis (PXIe-1082). This eight-core controller provides parallel processing capabilities of up to 16 simultaneous tasks with hyper-threading and as such is well suited to the task of solving multiple FEA models in parallel. Data acquisition was performed using a 14-bit ADC (PXI-6133) for the linear transducer (SPS-L225-HALS manufactured by Honeywell) while acceleration data was acquired using a 24-bit IEPE ADC (NI-9234).

2.3. Finite element analysis modeling

In this work, a simplified 1-D FEA model using Euler–Bernoulli beam elements with a linear interpolation function was constructed with a fixity (no displacement or rotation) on the left-hand side and a movable roller supporting the beam as depicted in Fig. 3. The boundary conditions were applied by decimating the rows and columns associated with the boundary conditions of the system in the global mass (**M**) and stiffness (**K**) matrices. To expand, for the simplified case presented in Fig. 3(b) the columns and rows associated with u_1 and u_2 are removed to enforce the boundary condition at the fixity while the column and row associated with u_5 are removed to enforce the boundary condition at the roller. Thereafter the updated mass and stiffness matrices can be used directly for modal analysis as described in Section 2.4. This simplified FEA model formulation was selected to reduce the computational cost related to constructing and enforcing the boundary conditions in the FEA model. To calibrate the FEA model, the beam's stiffness and length where adjusted by comparing the beams measured resonant frequency with the frequency computed using an FEA with 1000 nodes. These updates where performed using a particle swarm optimization algorithm.

2.4. Modal analysis

Modal analysis is used to calculate the natural frequencies of the structure and can be derived from the equation of motion for the DROPBEAR beam under free vibration:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = 0 \tag{1}$$

The damping term ($\mathbf{C}\dot{x}$) can be ignored as its effect on the frequency of vibration is insignificant. To expand, experimental testing showed that for the case where the roller is positioned 48 mm away from the support the critical damping ratio

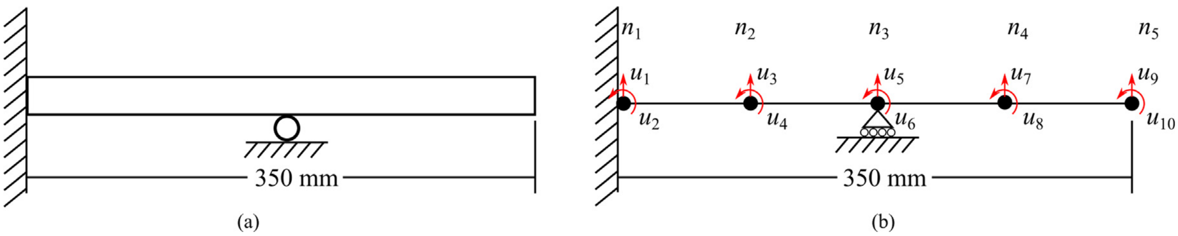

Fig. 3. Diagram of the FEA model used in this work showing: (a) a schematic representation of the DROPBEAR; and (b) a five node FEA of the type used in this work.

(ζ) is 0.0076. The theoretical resonant frequency of the beam can be computed assuming an impact excitation on the beam that equally excites all the frequencies of the beam. Calculations show that the computed resonant frequency is reduced by only 0.005% from that of the undamped natural frequency. Therefore, a simplified expression of the equation of motion that ignores the damping term can be used and is expressed as:

$$\mathbf{M}\ddot{x} + \mathbf{K}x = 0 \tag{2}$$

where gravity does not affect the equation of motion. The harmonic solution $x = \Phi \sin(\omega_n t)$ is assumed where Φ represents the mode shapes of the solution and ω_n is the undamped natural frequency of the structure. Inserting the assumed solution into Eq. 2 and simplifying the resulting expression yields:

$$(-\boldsymbol{\omega}_n^2\mathbf{M} + \mathbf{K})\boldsymbol{\Phi} = 0 \tag{3}$$

where $\boldsymbol{\Phi} = 0$ is a trivial solution. Therefore, a solution of interests is one where the natural frequencies ω_n and modes Φ must satisfy the generalized eigenvalue problem:

$$\mathbf{K}\boldsymbol{\Phi} = \lambda\mathbf{M}\boldsymbol{\Phi} \tag{4}$$

where $\boldsymbol{\lambda} = \boldsymbol{\omega}_n^2$. In this configuration $\boldsymbol{\lambda}$ is a diagonal matrix of eigenvalues and $\boldsymbol{\Phi}$ are the associated eigenvectors where $\omega_n = \sqrt{\lambda}$. Therefore, the undamped natural frequency of the system can be obtained by solving the generalized eigenvalue problem and identifying the lowest valued eigenvalue.

3. Methodology

The proposed millisecond error minimization model updating algorithm is diagrammed in Fig. 4 and described in what follows. The proposed algorithm is separated into two main portions that run in parallel, an experimental and an analytical portion. The experimental portion consists of an accelerometer mounted at the end of the beam that collects acceleration data. This data is then windowed with a Hanning window to prepare the data for further processing. Thereafter, a Fast Fourier Transform (FFT) of the windowed data is computed and the resonant frequency of the system is experimentally extracted. Running in parallel with the experimental portion, the analytical portion of the code continuously constructs a set of FEA models by sampling without replacement model parameters from a continuously updated parameter pool. For this introductory work, the sample parameter consists of the roller location of the DROPBEAR while the pool is modeled as a Gaussian distribution where the current roller location (i.e. state) is the center value of the distribution and the probability of sampling a new roller location is quantified by the Gaussian distribution. To expand, if the current location of the roller was determined to be at FEA node n, the probability of sampling FEA node n would be the highest, followed by $n \pm 1, n \pm 2, n \pm 3$, etc. Using modal analysis, the set of FEA models can be solved to obtain the undamped natural frequencies for each FEA model. Thereafter, the FEA model that best represents the system can be identified through quantifying the norm-2 error between the experimentally obtained resonant frequency and the undamped natural frequencies computed from the set of FEA models. After completing an iteration of the proposed algorithm and estimating the current state of the system,

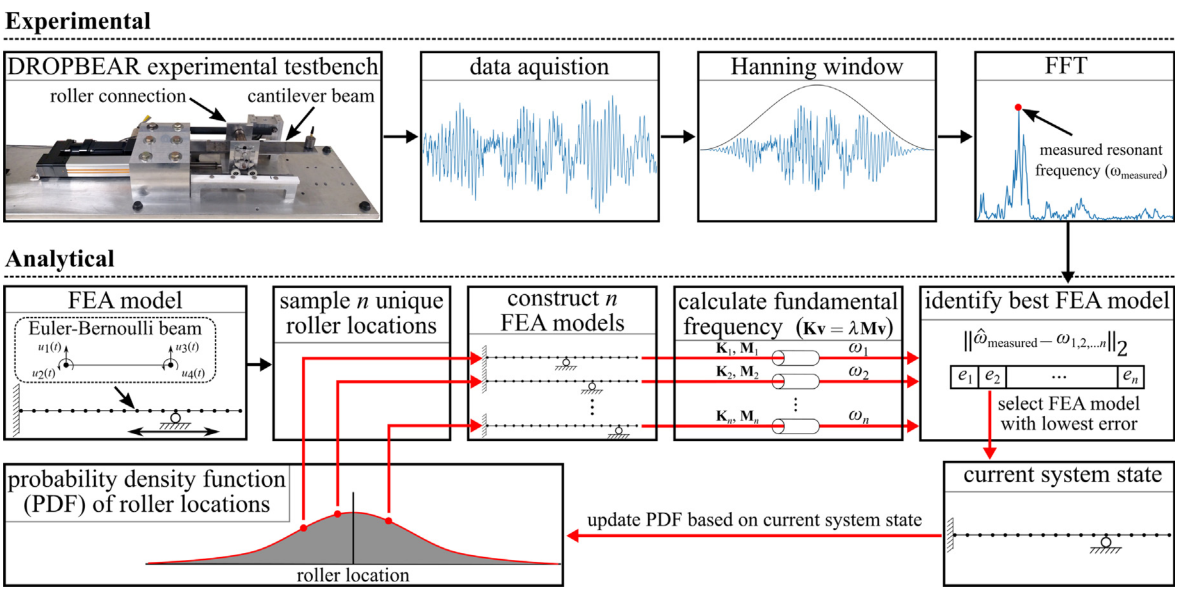

Fig. 4. Flowchart of the millisecond error minimization model updating algorithm utilized in this work.

the Gaussian distribution is reset such that the estimated roller location is the center of the distribution for the successive iterations. This process is repeated indefinitely to track the condition of the structure. A key consideration is that the structure must have a source of near consistent excitation in order for the proposed millisecond model updating algorithm to properly function. In this work, constant excitation for the beam comes from the roller movement. The test profile used in this work was designed to ensure that the beam maintained a sufficient level of vibration throughout the entire test profile.

A delay is present between the movement of the roller and estimated roller position that is correlated to how long it takes the dynamics of the system to propagate through the windowed data that is used for computing the FFT. The amount of data required by the FFT to accurately determine the resonant frequency of the structure without causing a prolonged delay in the algorithm is proportional to the dynamics of the monitored system. For the experimental investigation carried out in this work, a data set that consists of the most recently sampled 5000 acceleration data points was found to be appropriate. 5000 data points correspond to the acceleration data from the 198 ms before the current time. A level of uncertainty of the currently estimated roller location can then be computed by taking the standard deviation of all the estimated roller locations that were computed during the previous 198 ms. This uncertainty quantification helps to provide additional information that can be used for anomaly detection as well as to provide additional information to be used for the elimination of outliers in future work.

The proposed millisecond model updating algorithm has two user-selectable parameters (number of nodes in the FEA model and the number of FEA models solved in parallel) that must be set by the practitioner in order to deploy this algorithm in an operational environment. Moreover, there are two objective parameters (iteration time and estimation error) that the practitioner must try to minimize. There does not exist a single solution that simultaneously optimizes both parameters for both the considered objectives, however, this multi-objective optimization problem can be formulated to form a single objective optimization function whose solutions can be compared in a design space to select an optimal algorithm configuration [22]. Various methods can be used to find these solutions, this work utilizes a straightforward linear scalarization approach [23]. This linear combination method identifies the minimum value for a weighted combination of objectives, resulting in an optimal solution. The benefit of this approach is it allows for trade-offs to be made between the two objectives by altering the user-defined scalarization factor, α, thereby increasing the usability of the optimization function in defining the optimal algorithm configuration parameters ($\mathbf{P}$). $\mathbf{P}$ is constituted of p_{nodes} and p_{models} that represent the number of nodes in the FEA model and the number of FEA models solved in parallel, respectively, where $\mathbb{P}$ is the parameter search space. The single objective optimization problem can be formulated as:

$$\underset{\mathbf{P}}{\text{minimize}}\,\text{fit} = (1-\alpha)\frac{e(\mathbf{P})}{e'} + \alpha\frac{t(\mathbf{P})}{t'}\text{subject to }\mathbf{P} = [p_{\text{nodes}}, p_{\text{models}}] \in \mathbb{P} \tag{5}$$

where e represents the error in estimation and e' the maximum desired error. Moreover, t and t' define the iteration time and maximum desired iteration time, respectively. The primed values, e' and t', are factors used for normalizing e and t. This optimization problem can be converted to an error value minimization problem for $\alpha = 0$, or a, iteration minimization problem for $\alpha = 1$. Selection of an appropriate value for α is based on the system to be monitored and the characteristics of the dynamic environment in which it operates. For this preliminary work a value of $\alpha = 0.5$ is utilized along with e' = 10 mm and t' = 5 ms.

4. Results and discussion

This section reports the results and discussions from the numerical simulations and experiments. For the experimental campaigns, all results were computed in real-time using the computational resources described in Section 2.2 while the numerical verification was performed using algorithm timing achieved in the experimental campaign.

4.1. Numerical verification

Results for the numerical verification of the proposed millisecond model updating algorithm are presented in Fig. 5. These numerical investigations were developed to demonstrate the algorithm's response in two key areas, accuracy and convergence time. To isolate these responses, the algorithm iteration time is set to 5 ms for consistency and the true roller locations were obtained using a 1000 model FEA solved in the frequency domain. By solving the FEA in the frequency domain the time delay caused by the FFT as discussed previously is not present in these plots. For clarity, these simulations were done without any artificial noise and the sampled roller locations are constrained such that the current and adjacent FEA nodes are sampled for each successive iteration of the algorithm. Fig. 5(a) demonstrates the benefit of increasing the number of sampled nodes in the FEA model in relation to the accuracy of the estimated roller position. In this plot, each "step" in the estimated roller location is a function of the algorithm selecting the next FEA node as the roller position progresses with time. Fig. 5(a) shows that more nodes in the FEA results in greater accuracy as there is generally an FEA node closer to the real roller location. Fig. 5(b) reports the decrease in convergence time after a roller movement (e.g., unmodeled high-rate dynamic event) that is obtained by increasing the number of FEA models solved in parallel. As expected, increasing the number of FEA models solved in parallel reduces the convergence time for the algorithm. This is because more FEA models solved

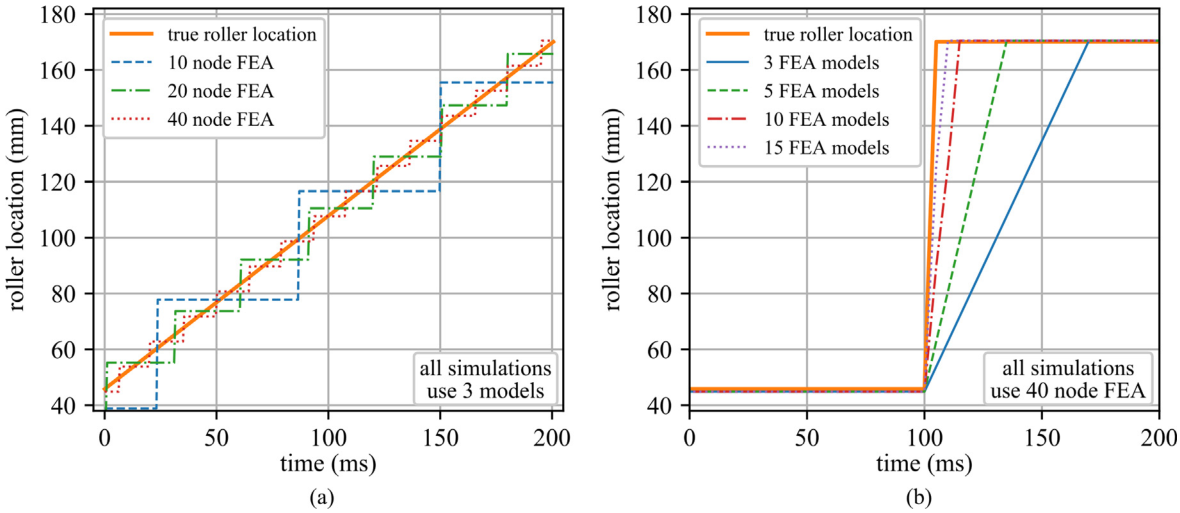

Fig. 5. Numerical verification for the millisecond model updating algorithms showing: (a) algorithms response for an increasing number of FEA nodes for a linear movement of the roller; and (b) for an increasing number of FEA models for a step change in the roller location.

in parallel enables the algorithm to search a wider set of roller locations and therefore test FEA configuration (e.g., pin locations) that are further away from the current state. As expected, searching over a wider space reduces the number of iterations needed to find the new state of the structure.

4.2. Real-time implementation

Algorithm timing results for a selected algorithm configuration with three 40 node FEA models extracted from the rising edge of a square function of the roller test profile presented in Fig. 6 where 0 ms is considered the current time. The temporal acceleration data used in the current iteration of the algorithm is annotated in Fig. 6(a) and consists of the most recent 5000 data points and corresponds to the data from the last 198 ms. Fig. 6(b) reports the timing associated with distinct parts of the algorithm where the majority of the time is devoted to solving the generalized eigenvalue problem in the modal analysis problem for each the three FEA models running in parallel. The time allocated to loading the data from the buffer and computing the FFT of the windowed data is relatively short when compared to the computational costs associated with the FEA models. The delay between the roller movement and the estimated roller location is annotated in Fig. 6(c) as is the 3σ level of uncertainty of the estimated roller location that was computed using the moving 198 ms window of the estimated roller locations. The delay between the movement of the roller and the algorithm registering this movement is caused by the time it takes for the acceleration data to propagate into the 198 ms window such that the signal related to the new roller position is the dominant signal computed by the FFT. To expand, a theoretical instantaneous movement of the roller would cause a direct change in the measured frequency of the beam. However, this new measured signal (and its associated frequency) would not be the dominant frequency in the 198 ms window until enough of the new signal has displaced the old signal from the window. The delay between the real and measured state is further investigated in Fig. 7 where Fig. 7(a) reports the measured frequency for the second sinusoidal roller movement in the test profile. No delay is present in the frequency space presented as the current configuration of the algorithm is capable of tracking the measured frequency of the system. However, when considering the measured roller movement as displayed in Fig. 7(b) the delay between the measured and estimated roller location is present as expected. These results further demonstrate that the delay between the measured roller location and the estimated roller location is caused by the time required for the dynamics of the system to propagate through the 198 ms moving windowed data set on which the FFT is computed.

Real-time model updating results are presented in Figs. 8–12. First, Fig. 8 reports the measured and estimated roller location for three algorithm configurations with 10, 20, and 40 nodes respectively where each algorithm configuration utilized three FEA models solved in parallel. As expected, an increase in the number of FEA nodes results in increased accuracy in the estimated roller location. This increase in accuracy is quantified by the mean absolute error (MAE) computed over the entire test profile and the percent error calculated as the MAE over the length of the beam (350 mm). These values are annotated in the top-left inset of each plot in Fig. 8. Additionally, these insets report the iteration time for each algorithm configuration. As expected, with an increase in the number of FEA nodes the iteration time increases while the error in the estimated roller location decreases. Experimental results achieved here demonstrate that the proposed millisecond model updating algorithm can successfully track the DROPBEAR's changing boundary condition. Using an FEA model with 10 nodes results in an accuracy of 8.9% updated every 0.32 ms. Moreover, an FEA model with 20 nodes achieves an accuracy of 3.9% updated every 0.86 ms while an FEA model with 40 nodes achieves an accuracy of 2.9% updated every 4.04 ms. The reported results

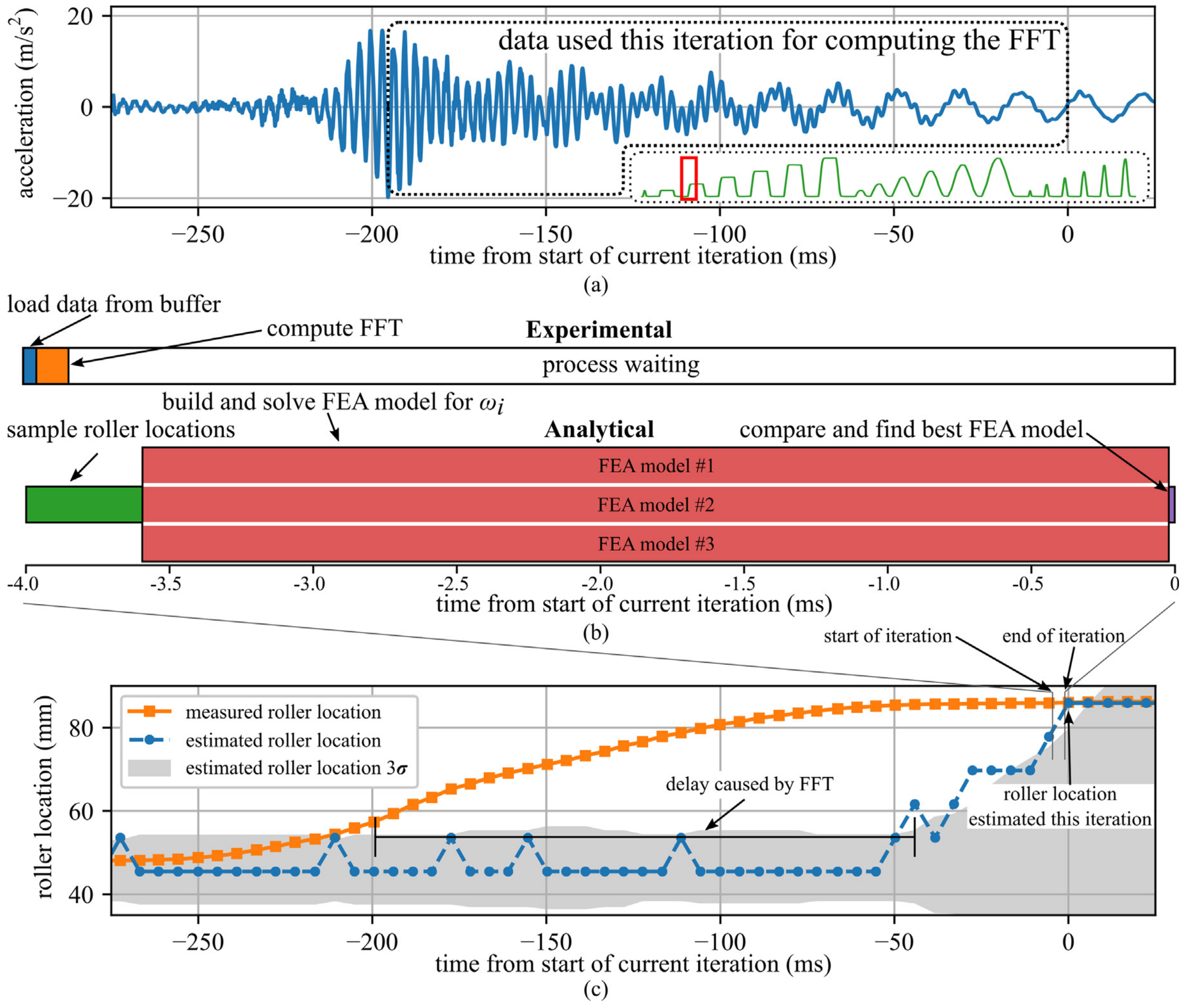

Fig. 6. Algorithm timing results for a 40 mm roller movement for an FEA model with 40 nodes sampled three times from the PDF showing the: (a) acceleration data; (b) timing of algorithm components; and (c) the estimated roller locations.

also help demonstrate that three FEA models solved in parallel are capable of successfully tracking the dynamics of the system (i.e. the movement of the roller) and as such an increase in the number of models solved in parallel does not provide an increase in the precision of the estimated roller location.

To further investigate the algorithm's response in terms of accuracy and timing, an experimental investigation performed that tested every combination of FEA nodes from 10 to 40 and number of FEA models from 3 to 15; these results, along with the results from the multi-objective optimization of algorithm parameters, are presented in Fig. 9. The experimental investigation is limited to 15 FEA models as 15 FEA models can be solved in parallel without accruing a large increase in the computation time as the controller only possesses 16 compute cores and one compute core is required for the experimental portion of the algorithm as diagrammed in Fig. 6(b). As expected, an increase in the number of nodes in the FEA model results in a decrease in the MAE as expressed in Fig. 9(a). However, due to the limited velocity of the roller (250 mm/s) an increase in the number of FEA models solved in parallel does not result in a noticeable increase in accuracy after three FEA models. The aliasing represented as peaks and valleys in Fig. 9(a) along the number of FEA models axis is a function of the changing FEA location of the nodes aligning up with the static FFT bins in the frequency domain. The effect of aliasing on the error value of the estimated roller position reduces with an increase in the number of nodes in the FEA models. Moreover, an increase in either the number of nodes in the FEA model or the number of FEA models solved in parallel results in an increase in the iteration time as presented in Fig. 9(b). The red lines in Figs. 9(a) and (b) are provided to express the relation of the results to the normalization factors (e' and t') used in Eq. 5. Results from the multi-objective optimization are shown in Fig. 9(c) where the algorithm configuration with the lowest algorithm design space fit achieved using the values $\alpha = 0.5, e'$ = 10 mm, and t' = 5 ms is annotated. The transparent plane added to Fig. 9(c) provides a reference where all the points below the plane fail to meet the combined design criteria of 1. To expand, their combined e and t values are

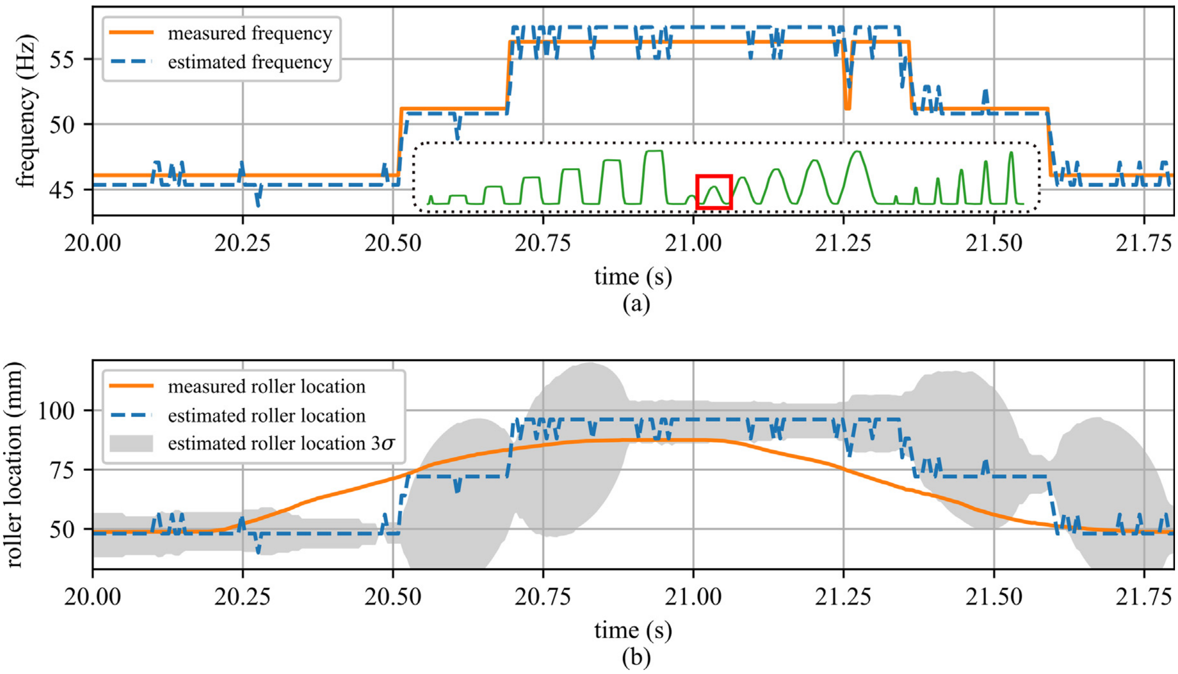

Fig. 7. Investigation of delay showing the: (a) measured vs estimated frequency for the second sinusoidal input; and (b) measured vs. estimated roller location.

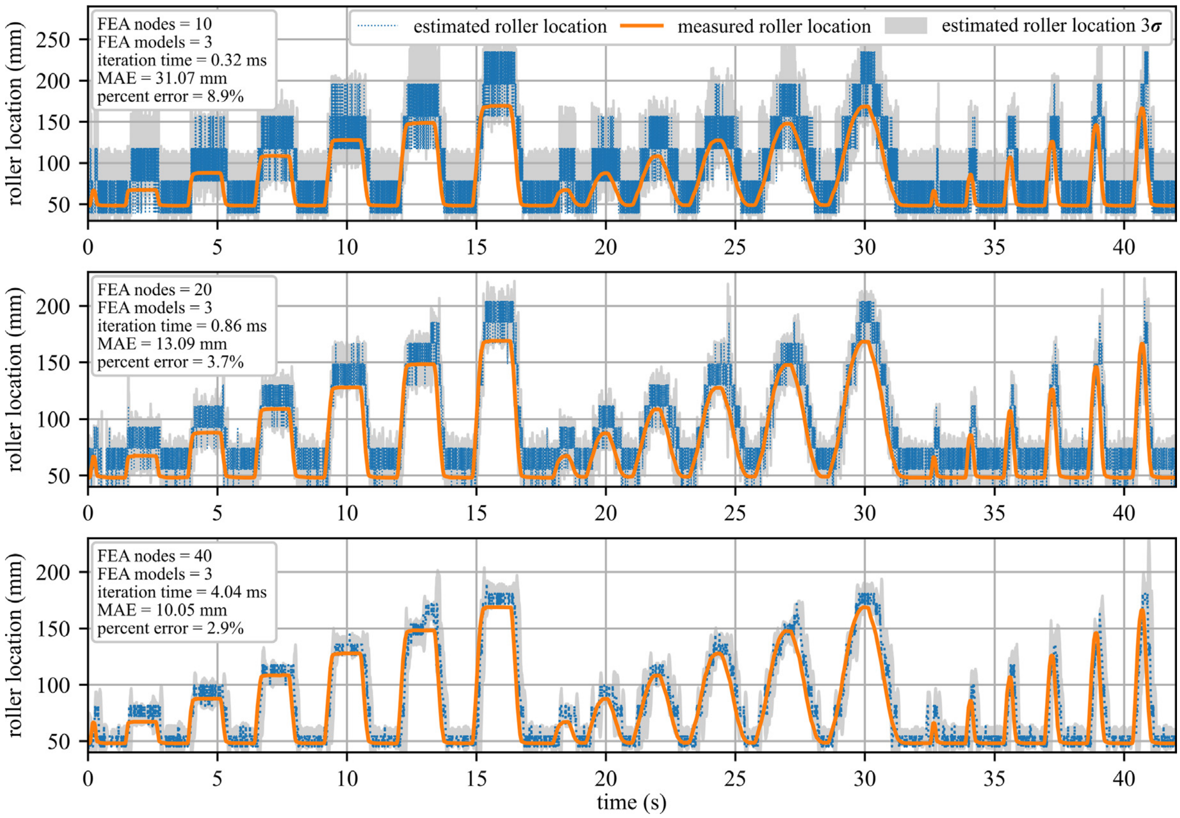

Fig. 8. Temporal results for three model configurations with 10, 20, and 40 nodes in the FEA where each configuration has three FEA models solved in parallel.

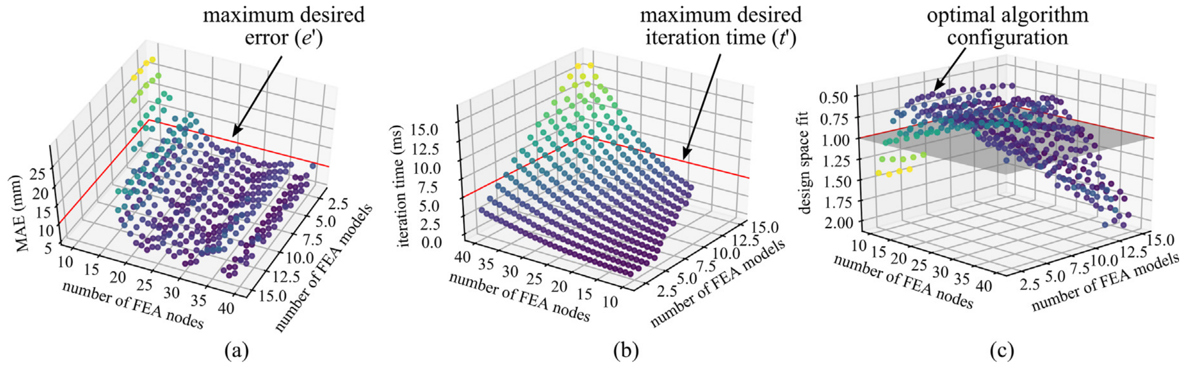

Fig. 9. Results from 233 automated tests showing the: (a) mean absolute error for each configuration; (b) average iteration time for each configuration; and (c) the design space computed for the current DROPBEAR configuration with the optimal algorithm configuration annotated. Note the reversal of the axes to facilitate plotting.

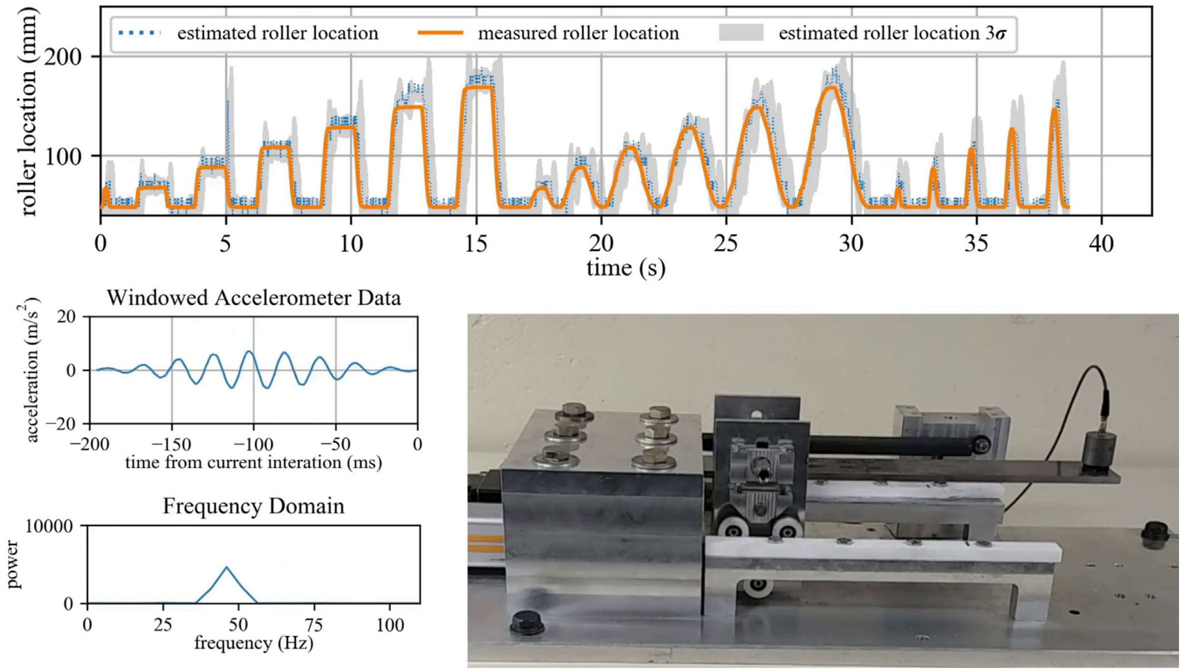

Fig. 10. Real-time video of the millisecond model updating algorithm for a model configuration with three FEA models each consisting of 40 nodes. This video displays the measured and estimated roller state (top), the Hanning windowed acceleration data and the FFT computed frequency data (bottom-left), and the DROPBEAR test bench (bottom-right). The video file is included as supplementary material in the online version of this article.

greater than the prescribed limits set by e' and t'. The optimal parameter configuration was found to be an FEA model with 16 nodes and five FEA models solved in parallel. These parameters resulted in a design space fit of 0.49, an MAE of 8.21 mm, and an iteration time of 0.83 ms. Fig. 10 presents a real-time video of the estimated roller position calculated using the millisecond model updating algorithm configured with three FEA models each consisting of 40 nodes. The video file is included as supplementary material in the online version of this article. Additionally, the acceleration data with the applied Hanning window and its computed frequency domain are presented. For the video, all results were calculated in real-time during the test with an iteration time of 4.04 ms while the graphics were rendered in post-processing.

To further investigate the proposed millisecond model updating algorithm, two special cases where investigated where each case uses an FEA model with 40 nodes and three FEA models solved in parallel. The first case consisted of an impact event imparted on the beam during testing (using a 6.33 mm ball bearing) and is presented in Fig. 11. This case demonstrates that the frequency-based error minimization technique used in the proposed millisecond model updating algorithm provides a level of robustness in terms of tracking the system state through an impact event. This would be expected as an ideal impact event will not alter the resonant frequency of the structure and therefore the frequency-based error minimization

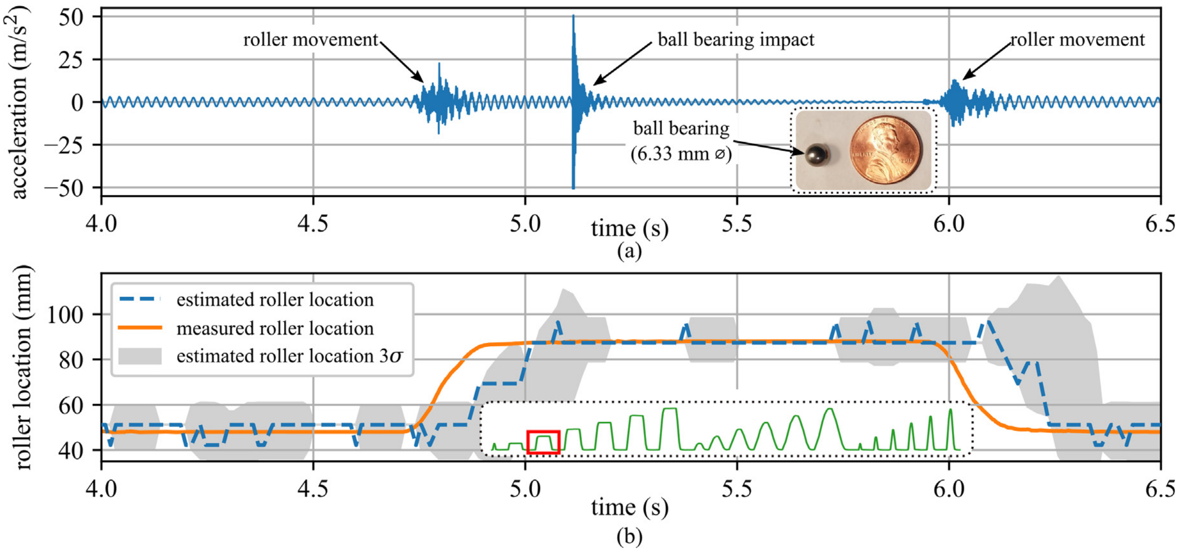

Fig. 11. Estimated roller location results for an impact on the beam that occurs directly following a roller movement showing: (a) the raw acceleration data and ball bearing in the inset; and (b) the estimated and measured roller location.

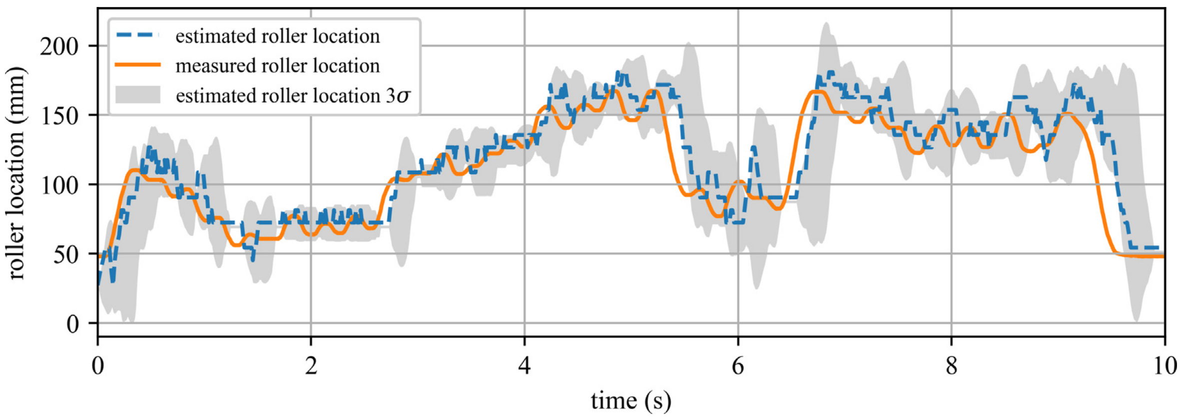

Fig. 12. Estimated roller location results for a stochastic movement of the roller.

technique would not be affected. Second, a stochastic temporal input for the roller location was investigated and is presented in Fig. 12. Here the capability of the proposed millisecond model updating algorithm to track the roller position is demonstrated. Over the entire test, an MAE of 13.08 mm was obtained. The stochastic validation demonstrates that once the delay caused by the length of the data set computed by the FFT is accounted for, the proposed millisecond model updating algorithm is capable of accurately tracking the roller location through a stochastic dynamic event.

5. Summary and conclusions

This paper proposed and validated a millisecond model updating algorithm for estimating the current state of a structural system in real-time. This model updating algorithm is specially formulated for structures experiencing unmodeled high-rate dynamics and as such cannot rely on precalculated data sets or offline training. Therefore, all computations were computed on-the-fly while the structure was experiencing dynamic events. For the proposed algorithm, model updating was performed using a frequency-based error minimization technique that compared the measured resonant frequency of a structure to those obtained by modal analysis for a set of Finite Element Analysis (FEA) models with varying parameters solved in parallel where the FEA model with the highest level of agreement was considered to be the current state of the system. In constructing the FEA models, parameters are sampled without replacement from a dynamic pool that is updated with each successive iteration of the algorithm.

This work numerically and experimentally validated the proposed millisecond model updating algorithm using the Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) testbed at the Air Force Research Laboratory. For this work, a continuously changing boundary condition in the form of moving roller support was tracked in real-time. Numerical results demonstrated that an increase in the fidelity of the FEA model and the size of the search space (i.e. the number of FEA models solved in parallel) results in an algorithm formulation with increased precision in tracking the system state of the structure through dynamic events. Building on these results, the experimental validation demonstrated that the proposed millisecond model updating algorithm was capable of operating at the timescale required by high-rate dynamic events (less than 10 ms). To expand, for the dynamic test profile considered, experimental results demonstrated that when three 40-node FEA models were solved in parallel the location of the roller on the DROPBEAR could be tracked and updated every 4.04 ms with an accuracy of 10.05 mm computed as the Mean Absolute Error (MAE) over the entire test profile. When considering the 350 mm beam, an accuracy of 2.9% is achieved. Experimental results demonstrated that the proposed algorithm was capable of tracking the position of the roller during an impact load imparted on the beam. Furthermore, the proposed algorithm configured with three 40-node FEA models solved in parallel demonstrated it was capable of tracking a stochastic roller movement with an accuracy of 13.08 mm (3.7%), computed as the MAE over the ten-second stochastic test.

The validated millisecond model updating algorithm presented in this research advances the field of real-time model updating for structures Experiencing unmodeled high-rate dynamic events by: 1) formulating an algorithm that does not rely on precalculated data sets or offline training; 2) validating that the roller position of the DROPBEAR could be accurately tracked with an algorithm interval time of less than 10 ms; 3) demonstrating the robustness of the algorithm by tracking the system state through impact loadings and under stochastic inputs.

CRediT authorship contribution statement

Austin Downey: Conceptualization, Methodology, Validation, Investigation, Formal analysis, Writing - original draft. **Jonathan Hong:** Conceptualization, Methodology, Validation. **Jacob Dodson:** Conceptualization, Methodology, Funding acquisition, Project administration. **Michael Carroll:** Software, Validation, Investigation. **James Scheppegrell:** Software.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ymssp.2019.106551.

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